EXAMPLE 5 ➤ Show numerically that $\log 5^3 = 3 \log 5$. Explain how this property can be derived from the log of a product property.

SOLUTION

$\log 5^3 = \log 125 = 2.0969\ldots$

$3 \log 5 = 3 \cdot 0.6989\ldots = 2.0969\ldots$ Calculate without rounding.

$\therefore \log 5^3 = 3 \log 5$ Combine like terms.

This equality derives from the product of a log property because

$\log 5^3 = \log(5 \cdot 5 \cdot 5)$

$= \log 5 + \log 5 + \log 5$ The log of a product equals the sum of the logs of the factors.

$= 3 \log 5$ Combine like terms.

Example 6 shows you how to prove algebraically that the logarithm of the product of two numbers equals the sum of the logarithms of the factors.

EXAMPLE 6 ➤ Prove algebraically that $\log xy = \log x + \log y$.

SOLUTION *Proof*

Let $c = \log x$ and let $d = \log y$.

Then $10^c = x$ and $10^d = y$. By the definition of logarithm.

$xy = 10^c = 10^d$ Multiply x times y.

$xy = 10^{c+d}$ Add the exponents. Keep the same base.

$\log xy = c + d$ The logarithm is the exponent of 10.

$\therefore \log xy = \log x + \log y$, Q.E.D Substitute for c and d.

In Problems 45 and 46, you will prove the other two properties of base-10 logarithms algebraically.

Example 7 shows you how to use the properties of logarithms to find the number that goes in the blank.

EXAMPLE 7 ➤ Use the properties of logarithms to find the number that goes in the blank: $\log 3 + \log 7 - \log 5 = \log$ __?__. Check your answer numerically.

SOLUTION

$\log 3 + \log 7 - \log 5 = \log \frac{3 \cdot 7}{5} = \log 4.2$

$\therefore$ 4.2 goes in the blank.

CHECK

$\log 3 + \log 7 - \log 5 = 0.6232\ldots$ By calculator.

$\log 4.2 = 0.6232\ldots$

$\therefore \log 3 + \log 7 - \log 5 = \log 4.2$